

TD de Logique n° 3

Dédution Naturelle

Exercice 1 Montrez que $\Delta, A \vdash A$ est valide.

Exercice 2 Montrer que si les séquents $\Delta \vdash A$ et $\Delta \vdash B$ sont valides alors le séquent $\Delta \vdash A \wedge B$ est valide.

Exercice 3 En utilisant $\vdash_{DN_{prop}}$, montrez les propriétés suivantes:

1. $\vdash (\mathbf{p} \vee \mathbf{q}) \rightarrow (\mathbf{q} \vee \mathbf{p})$
2. $\vdash (\mathbf{p} \vee (\mathbf{q} \wedge \mathbf{r})) \rightarrow (\mathbf{p} \vee \mathbf{q}) \wedge (\mathbf{p} \vee \mathbf{r})$
3. $\vdash ((\mathbf{p} \rightarrow \mathbf{q}) \wedge (\mathbf{p} \rightarrow \neg \mathbf{q})) \rightarrow \neg \mathbf{p}$
4. $\vdash \neg(\mathbf{p} \vee \mathbf{q}) \rightarrow (\neg \mathbf{p} \wedge \neg \mathbf{q})$
5. $\vdash (\neg \mathbf{p} \vee \mathbf{q}) \rightarrow \mathbf{p} \rightarrow \mathbf{q}$
6. $\vdash (\mathbf{p} \rightarrow \mathbf{q}) \rightarrow (\neg \mathbf{p} \vee \mathbf{q})$ (*On pourra s'aider du tiers exclus, vu en cours*)
7. $\vdash \neg(\mathbf{p} \wedge \mathbf{q}) \rightarrow (\neg \mathbf{p} \vee \neg \mathbf{q})$
8. $(\mathbf{p} \rightarrow \neg \mathbf{q}), ((\neg \mathbf{q} \vee \mathbf{r}) \rightarrow \neg \mathbf{s}), \mathbf{p} \wedge \mathbf{t} \vdash \neg \mathbf{s}$
9. $\neg(\mathbf{p} \wedge \mathbf{q}), \mathbf{p} \vdash \neg \mathbf{q}$
10. $(\mathbf{p} \wedge \mathbf{q}) \rightarrow \neg \mathbf{r} \vdash \mathbf{r} \rightarrow (\mathbf{p} \rightarrow \neg \mathbf{q})$
11. $(\mathbf{p} \rightarrow (\mathbf{q} \vee \mathbf{r})), (\mathbf{q} \rightarrow \mathbf{r}) \vdash (\mathbf{p} \rightarrow \mathbf{r})$
12. $\mathbf{p} \rightarrow \mathbf{q} \vdash \neg \mathbf{q} \rightarrow \neg \mathbf{p}$
13. $\mathbf{p} \rightarrow (\mathbf{q} \rightarrow \mathbf{r}) \vdash \mathbf{p} \wedge \mathbf{q} \rightarrow \mathbf{r}$
14. $\vdash \mathbf{p} \wedge (\neg \mathbf{p} \vee \mathbf{q}) \leftrightarrow (\mathbf{p} \wedge \mathbf{q})$
15. $\vdash (\mathbf{p} \rightarrow \mathbf{q} \wedge \mathbf{r}) \rightarrow (\mathbf{p} \wedge \mathbf{q} \leftrightarrow \mathbf{p} \wedge \mathbf{r})$