From λ-terms to MELL Proof-Nets

Recall that...

Two forms for atomic formulae : p and $\underline{p}$.

Negation of formulae is defined as follows :

$$
\begin{array}{llllll}
p^{\perp} & := & \underline{p} & \underline{p}^{\perp} & := & p \\
(A \parr B)^{\perp} & := & A^{\perp} \otimes B^{\perp} & (A \otimes B)^{\perp} & := & A^{\perp} \parr B^{\perp} \\
(?A)^{\perp} & := & !A^{\perp} & (!A)^{\perp} & := & ?A^{\perp}
\end{array}
$$

Reduction rules and equations for MELL Proof-Nets

See

`http://www.pps.univ-paris-diderot.fr/~kesner/enseignement/mpri/ll/RE-MELL-proofnets.pdf`

Reduction relation for MELL Proof-Nets

Let us consider :

$$
\begin{array}{lcl}
E & = & A \cup B \\
CE & = & \{Ax\text{-}cut, \parr\text{-}\otimes, w\text{-}b, d\text{-}b, c\text{-}b, b\text{-}b\} \\
R & = & CE \cup \{\mathtt{U}, \mathtt{V}\}
\end{array}
$$

The reduction relation on MELL proof-nets is generated by the reduction rules R and congruence axioms E :

$$p \rightarrow_{R/E} p' \text{ iff } \exists p_1, p_2 \; p \sim_E p_1 \rightarrow_R p_2 \sim_E p'$$

Termination properties of proof-nets

(Girard)

The reduction system generated by the reduction rules CE is SN.

(DiCosmo-Guerrini)

The reduction system generated by the reduction rules $CE \cup \mathtt{V}$ modulo the axioms E is SN.

(Polonovski)

The reduction relation R/E is SN.

From simply typed λs-terms to MELL Proof-Nets

Translate types and type derivations as follows :

`http://www.pps.jussieu.fr/~kesner/enseignement/mpri/ll/lambdas-proofnets2.pdf`

Translating λs-Reduction on Typed Terms

Theorem [From λs to MELL] Let $\Gamma \vdash_{\lambda\mathsf{s}} t : A$ and $t \rightarrow_{\lambda\mathsf{s}} t'$, then $T(\Gamma \vdash_{\lambda\mathsf{s}} t : A) \rightarrow^*_{R/E} C[T(\Gamma' \vdash_{\lambda\mathsf{s}} t : A)]$ for some $\Gamma' \subseteq \Gamma$ and some MELL-context made only of weakenings.

- Which λs-steps are strictly/weakly translated ?
- How SN for λs-typed terms can be concluded from SN for MELL Prof-Nets ?

The σ-equivalence in λ-calculus

$$
\begin{array}{llll}
(\lambda x.\lambda y.U)V & \equiv_{\sigma_1} & \lambda y.(\lambda x.U)V & \text{if } y \notin FV(V) \\
(\lambda x.UV)W & \equiv_{\sigma_2} & ((\lambda x.U)W)V & \text{if } x \notin FV(V)
\end{array}
$$

Lemma If $t \equiv_\sigma t'$, then $t \equiv_\beta t'$.

Theorem [Regnier'90] If $t \equiv_\sigma t'$, then $\eta_\beta(t) = \eta_\beta(t')$.

The σ-equivalence in calculi with ES

$$\begin{array}{lcll} (\lambda y.U)[x/V] & \equiv & \lambda y.U[x/V] & \text{if } y \notin FV(V) \\ (UV)[x/W] & \equiv & U[x/W]V & \text{if } x \notin FV(V) \end{array}$$