Exercices

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1 Exercise 1

Give a rational expression for the following languages:
(1) $a^{-1}(bA^* \cup aabA^*)$
(2) $a^{-1}(A^*abaA^*)$
(3) $a^{-1}(aba)^*$

2 Exercise 2

Consider the automaton $\mathcal{A}$ represented in Figure 1.

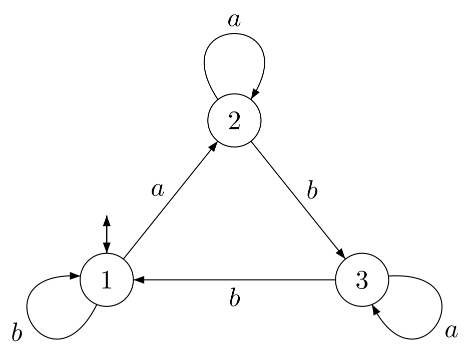

Figure 1: The automaton $\mathcal{A}$.

Give a rational expression for the language L recognized by $\mathcal{A}$.

3 Exercise 3

Compute the transition monoid M of the automaton $\mathcal{A}$ (Hint: you should find 12 elements). What are the idempotents of M ?
Is M an aperiodic monoid ? Is it commutative ?
Is L star-free ? Is it commutative ?

Solution

4 Exercise 1

(1) $a^{-1}(bA^* \cup aabA^*) = abA^*$
(2) $a^{-1}(A^*abaA^*) = A^*abaA^* + baA^*$
(3) $a^{-1}(aba)^* = ba(aba)^*$

5 Exercise 2

A rational expression for L is $(b + aa^*ba^*b)^*$. There are of course other solutions.

6 Exercise 3

The transition monoid M is

		1	2	3
*	1	1	2	3
*	a	2	2	3
	b	1	3	1
	ab	3	3	1
	ba	2	3	2
*	bb	1	1	1
	aba	3	3	2
	bab	3	1	3
*	bba	2	2	2
*	$abab$	1	1	3
*	$baba$	3	2	3
*	$bbab$	3	3	3

Relations $aa = a$ $abb = bb$ $bbb = bb$ $ababa = a$ $babab = b$ $bbaba = bbab$

The idempotents are $1, a, bb, bba, abab, baba, bbab$. The monoid M is not aperiodic since $(ab)^3 = ab$, but $(ab)^2 \neq ab$. Therefore, the language recognized by this automaton is rational but not star-free. It is not commutative since $ab \neq ba$ in M.