Exercises

October 21, 2008

Jean-Eric Pin

1 Exercise 1

Fill in the following table by YES or NO.

	Intersection	Complement	Morphisms	Product	Residuals
Rational languages are closed under					
Star-free languages are closed under					
Commutative languages are closed under					

2 Exercise 2

Let $A = \{a, b\}$. Indicate, for each of the languages L_1, L_2, L_3, L_4 whether it is rational, star-free or commutative.

(1) $L_1 = bA^*abA^* \cap A^*bbA^*$,

(2) $L_2 = \{u \in A^* \mid |u| \equiv 2 \bmod 5\}$

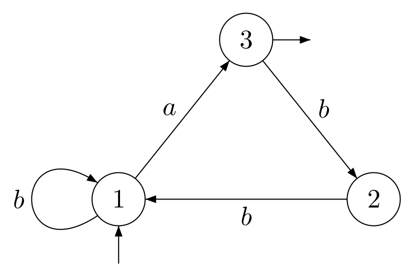
(3) $L_3 = (A^2)^*(a + bb)$,

(4) $L_4 = A^*(ab + ba)A*$.

Briefly justify your answers.

3 Exercise 3

Compute the transition monoid of the following automaton (Hint: you should find 12 elements).



Is this monoid aperiodic?

Is the language recognized by this automaton rational? Star-free? Commutative?

Solution

4 Exercise 1

Fill in the following table by YES or NO.

	Intersection	Complement	Morphisms	Product	Residuals
Rational languages are closed under	YES	YES	YES	YES	YES
Star-free languages are closed under	YES	YES	NO	YES	YES
Commutative languages are closed under	YES	YES	NO	NO	YES

5 Exercise 2

Let $A = \{a, b\}$. Indicate, for each of the languages L_1, L_2, L_3, L_4 whether it is rational, star-free or commutative.

(1) $L_1 = bA^*abA^* \cap A^*bbA^*$ is rational, star-free (a star-free expression for L_1 is $b\emptyset^c ab\emptyset^c \cap \emptyset^c bb\emptyset^c$) but not commutative since $babb \in L_1$ but $abbb \notin L_1$.

(2) $L_2 = \{u \in A^* \mid |u| \equiv 2 \bmod 5\}$ is rational (it is recognized by the finite monoid $\mathbb{Z}/5\mathbb{Z}$), but not star-free (its syntactic monoid is a group, hence it is not aperiodic), but it is commutative (since $\mathbb{Z}/5\mathbb{Z}$ is commutative).

(3) $L_3 = (A^2)^*(a + bb)$ is rational, but not star-free (since $L_3a^{-1} = (A^2)^*$ is not star-free), nor commutative, since $ba \in L_3$ but $ab \notin L_3$.

(4) $L_4 = A^*(ab + ba)A^*$ is rational, star-free and commutative

6 Exercise 3

The transition monoid M is

		1	2	3
$*$	1	1	2	3
	a	3	0	0
	b	1	1	2
$*$	aa	0	0	0
	ab	2	0	0
	ba	3	3	0
$*$	bb	1	1	1
$*$	abb	1	0	0
$*$	bab	2	2	0
$*$	bba	3	3	3
$*$	$babb$	1	1	0
$*$	$bbab$	2	2	2

Relations $\quad aa = 0 \quad aba = 0 \quad baa = 0 \quad bbb = bb \quad abba = a \quad bbabb = bb$

It is aperiodic since $1, aa, bb, abb, bab, bba, babb, bbab$ are idempotent, $a^2 = (ab)^2 = (ba)^2 = 0$ and $b^3 = b^2$. Therefore, the language recognized by this automaton is rational and star-free. It is not commutative since $ab \neq ba$ in M.