

Input- or output-unary sweeping transducers are weaker than their 2-way counterparts

Bruno Guillon

Revision details

First, I want to thank both referees for the careful review and their constructive comments. Most of the corrections and suggestions have been implemented in the revised version of the article, making it surely more readable. The few unimplemented comments are discussed in Section 1. The most relevant implemented corrections are detailed in Section 2.

1 Unimplemented suggestions and corrections

1.1 Lemma 4.3

[REF2] *Lemma 4.3:* — *The statement just says $O(|w|)$, but later the proof of Theorem 4.4 needs to refer to it using the more precise bound $c(|w|+1)$. Perhaps then this most precise bound should be included in the statement of the Lemma.*

We disagree with the referee. Indeed, the bound $c(|w| + 1)$ is no more precise than just saying $\mathcal{O}(|w|)$. We cannot easily modify the Lemma since no precise bound is stated.

1.2 Figure 2

[REF2] *Fig. 2a:* — *I believe an arrow from q to q' has not been printed. Similarly, Fig. 2b is missing two arrows.*

This is probably due to the pdf reader (or the printer). I opened the file with several reader (evince, zathura, mupdf, xpdf, Google viewer...), the arrows are correctly printed. I did not change anything, but I will check that they appear on the final version.

1.3 Proof of Theorem 4.2

[REF2] *p.17, l-6 and l-4:* — *The exponent r_e appears three times in these two lines. I think it should not be there.*

The exponent is well positioned but the notation was not introduced. For a set X and an integer p , the set X^p is the sum of p copies of X . This has been specified at the beginning of Section 3, and a recall has been introduced in the proof.

1.4 Proof of Lemma 5.2

[REF1] *p.21, l.-7* — *You probably mean "the language LR-PREFIX on input a^n has cardinality ..."*

As defined in Example 5.1, LR-PREFIX is a relation. We may thus speak of the language of outputs associated to some input w by LR-PREFIX, which is denoted $\text{LR-PREFIX}(w)$ according to the notation introduced in Section 2.1.

2 Implemented suggestion

In this section, we briefly describe how we implemented the most relevant corrections and suggestions.

2.1 Announce both results

[REF2] « *Title, abstract, and elsewhere: There are two results in this paper, one for unary output and one for unary input. But the author often writes (e.g., in the title) as if there is only one result. I understand that that result is more involved. But in announcing the paper, both results should be mentioned with equal strength —to say the least, it is intellectually much more pleasing to hear them both.* »

The title has been changed in order to announce both results. In the abstract, both results are now clearly presented at the same level and the introduction contains a more precise description of the second result (in particular in the outline paragraph).

2.2 Title

[REF2] *title* — *"Sweeping..." is syntactically incorrect here. We need a noun, e.g., "Sweepingness..." (which is ugly) or "A sweeping head..."*

As observed previously, the title has been entirely changed and the error no longer holds.

2.3 Abstract

[REF1] *abstract, line 5* — *I don't understand what it does mean that an alphabet should be reversely. This should be explained.*

[REF2] *abstract* — *"reversely": this is not the correct word here. You probably want to say "conversely", but this is again wrong. It is much better to write a full sentence.*

The last sentence has been changed for a more symmetrical formulation. The result is now clear.

2.4 Introduction

[REF1] *p.3, Table 1* — *Please explain how Table 1 has to be interpreted.*

A description of the table has been introduced in the introduction. Also, we added the input-unary line in order to summarize our two results as discussed above. For completeness, we filled the case of input-unary 2-way deterministic transducers by citing [2].

2.5 Outline

[REF2] *last par. of Section 1:* — *Perhaps the plan for Sections 4 and 5 could be more detailed here. E.g., see the outline given in my general comment above. With such level of detail, a reader that only reads Section 1 would still have learned the main points of this paper [Prop. 2.13, Thm 4.4, Lem 4.6, Lem 5.2].*

We have detailed the outline of paper, including the main steps of our proofs.

2.6 Proofs

2.6.1 Proof of Lemma 5.2

[REF1] « *in my impression the proof of Lemma 5.2 is not correct yet, since the author argues that the image of a transition is finite. This contradicts the definition of the author saying that the image of a transition may be a subset of a rational language, so in particular an infinite subset. Since Section 5 is the main contribution for the extension of the workshop*

version, this point has to be settled in any case. »

[REF1] p.21, l.-11 — "... the image of every input word is finite". This is a contradiction to Definition 2.4 where ϕ is defined as a mapping of δ into the set of non-empty rational subsets. Thus, I don't understand why $\phi(t)$ should be finite.

We checked the proof which was correct. The key point is that not all the transducers are considered rather only those which may accept the relation LR-PREFIX. Since, in the relation, each input is associated to a finite language, we can easily show that no transition with infinite associated production may be used during a successful run (otherwise, a infinite amount of outputs could be produced). Therefore, we may suppose that every transition is mapped into a finite set.

However, the proof were hard to read and some steps were probably not clear enough. It has been fully rewritten, in order to detail some points and to make it more readable.

2.6.2 Other proofs

[REF1] p.11, proof of Prop. 2.12. — You write "The formal proof ... is left to the reader." [...] It is strongly suggested to give more details of the construction. It should be the decision of the reader to skip details. Figure 3 is also somehow vague. You should explain the notion $1_{\mathbb{K}}$.

[REF2] Fig. 12: — The symbol K is used without definition. Also, all other figures use the vertical bar "|" instead of the semicolon ":".

[REF2] Also: — It would be good to enlarge this figure, as it is too small to read.

Figure 3 has been replaced by a plain text description of the construction.

[REF1] p.14, proof of Prop. 3.5 — It would be good to give more structure to the proof. "The minimal period divides p ." The minimal period of what, of X ? It is not clear to me why $p_i + \hat{p}_i$ should be in X . Give more details.

The whole proof has been revised, since many steps were unclear.

[REF1] p.15, proof of Lemma 3.9 — You should give more details for the extension to the general case which is not understandable in its present form. In the set X/r , where is x from?

[REF2] p.15 l-4: —

- In the def. of X/r the " N " should be " X ".
- In the expression rX/r^* , perhaps good to use parentheses: $r(X/r)^*$ —otherwise there is some confusion between $(X/r)^*$ and $(X^*)/r$.
- Also useful to observe that $\gcd(X/r) = 1$
- Good to use parentheses in the end of the line: $r(A \cup (t + N))$

The proof has been corrected and now additional details are given.

[REF1] p.12, l.-10 — "... is left to the reader". As said above, it is strongly suggested to give more details or to give at least a citation of "the famous characterization".

We added a citation to [3]. The proof is still omitted since it is basically another way to see semilinear sets.

2.7 Proposition 2.11

[REF2] p.9 l-10: — I am not sure what it means for Δ to be commutative. I understand that the text makes sense if we replace "commutative" with "unary". To use the word "commutative", we need to define it first.

[REF1] p.9, Proposition 2.11 — You should mention that Δ is commutative. What

about Σ ?

[REF2] *Prop. 2.11:* — The assumption that Δ is unary should be included in the statement of the Proposition.

The word “commutative” has been replaced by “unary”, and the assumption have been inserted in the statement of the proposition.

2.8 Definitions

[REF2] *p.6 par.5:* — Talking of ϕ being “single-valued” is a bit confusing. Technically, this is a function from transitions to sets. So, even if each of these sets have many elements, ϕ is still single-valued (it just happens that this one value is a set).

[REF1] *p.6, l.17* — Explain what single-valuedness means. Is it meant on runs or on inputs?

The definition of single-valuedness has been introduced precisely.

[REF1] *p.5, l.13* — “an accepting configuration has no successor.” You should explain that this is not a restriction.

Indeed it makes sense only for deterministic devices. The restriction has been removed from Definition 2.1, but it has been added in the definition of deterministic automata (and hence deterministic transducers).

2.9 Terminology

[REF2] *Section 3.1:* — Calling a set of numbers (subset of \mathbb{N}) “rational” may cause some confusion to some readers —a “rational number” is a very common phrase, with different meaning. Perhaps better to call the set “regular”.

We have implemented the suggestion, and we replaced every occurrences of rational by regular, when speaking of subsets of \mathbb{N} .

2.10 Automata description

[REF2] *Example 2.6, Proposition 4.5, Example 5.1:* — It’s always best if next to the state diagram you also give a description of the transducer in plain English.

We wrote a plain English description of the depicted automata at each time.

[REF2] *Fig. 4, caption, last line:* — The meaning of 0 and 1 here is respectively the empty string and the single-symbol string “a”; but they may easily be confused with the symbols 0 and 1. Perhaps a clarification is due.

The symbols 0 and 1 have been replaced by ϵ and a respectively.

2.11 Bibliography

[REF1] *p.2, l.1-2* — Give citations of the literature on these results.

[REF1] *Reference [1]* — Has the paper appeared in LNCS series? If so, this should be mentioned. What does “print: 1.” mean? Add the publisher.

[REF1] *Reference [5]* — Add the publisher.

[REF1] *Reference [10]* — “Frederic Servais” should be “Servais, F.”

[REF1] *Reference [13]* — “M. Shepherdson” should be “Shepherdson, J.C.”

[REF1] *Reference [7]* — Has the paper appeared in LNCS series? If so, this should be mentioned. What does “print: 1.” mean? Add the publisher.

done.

[REF1] *p.20, l.8* — Give more information on reference [11]. Where can the asymptotic behavior of p_n be found there? Wouldn't it be possible to refer to [3]? Give some page number where the result cited from [3] may be found. Give the simple computation that shows that p_n is in $\Omega(\dots)$ At the moment, the calculation is not clear to me since one has to be careful with the constants. Thus, it is proposed to give more details.

[REF1] *Reference [11]* — Give more information on this reference.

The proof has been changed in order to use Landau's function [4]. So, on one hand the proof is clearer, on the other hand we remove citations to Bach and Shallit (previously [3]) and to Fux (previously [11]).

3 Other changes

The collapse of the families of relations accepted by input-unary sweeping transducers and input-unary Hadamard relations were previously not given, though it was presupposed (e.g., Theorem 5.3). This has been fixed by adding Proposition 2.14, whose proof can be deduced from the work of Lombardy, [5].

We also add the citation to [1] in the introduction, since it is proved that the one-way definability of functional sweeping transducers is decidable.

References

- [1] BASCHENIS, F., GAUWIN, O., MUSCHOLL, A., AND PUPPIS, G. One-way definability of sweeping transducer. In *35th IARCS Annual Conference on Foundation of Software Technology and Theoretical Computer Science, FSTTCS 2015, December 16-18, 2015, Bangalore, India* (2015), pp. 178–191.
 - [2] CHOFFRUT, C. Sequences of words defined by two-way transducers. *Theoretical Computer Science* (May 2016).
 - [3] EILENBERG, S., AND SCHÜTZENBERGER, M.-P. Rational sets in commutative monoids. *J. Algebra* 13 (1969), 173–191.
 - [4] LANDAU, E. Über die Maximalordnung der Permutationen gegebenen Grades. *Arch. der Math. u. Phys.* (3) 5 (1903), 92–103.
 - [5] LOMBARDY, S. Weighted two-way automata. In *Seventh Workshop on Non-Classical Models of Automata and Applications - NCMA 2015, Porto, Portugal, August 31 - September 1, 2015. Proceedings* (2015), pp. 37–47.
-

4 Other comments

4.1 Referee 1

- [REF1] *resume, line 3* — "transducterus" should be "transducteurs"
- [REF1] *p.1, l. 4-5* — "In the automata theory" sounds strange.
- [REF1] *p.1, l.8* — "the two-tapes" should be "the two tapes".
- [REF1] *p.1, l.9* — "an output (write-only) tapes" should be "an output (write-only) tape"
- [REF1] *p.2, l.5* — Give the idea of the construction of this generalization.
- [REF1] *p.2, l.14* — I don't understand "A big departure". Probably "A big difference" is meant.
- [REF1] *p.2, l.23* — "nonempty" should be "non-empty". This has to be checked for the complete text.
- [REF1] *p.2, l.26* — "it is equivalent" should be "is equivalent".
- [REF1] *p.2, l.31* — "is usual in the automata theory" sounds strange.
- [REF1] *p.2, l.35* — Explain what "automaton component" means.
- [REF1] *p.2, l.40* — Reformulate the sentence "This highlights a link with another kind of restrictions: the input head move restrictions".
- [REF1] *p.3, l.6* — "This prove" should be "This proves".
- [REF1] *p.3, l.19* — "nonnegative" should be "non-negative". This has to be checked for the complete text.
- [REF1] *p.3, l.21* — "it as function" should be "it as a function".
- [REF1] *p.3, l.25* — "the sweeping transducers" should be "sweeping transducers".
- [REF1] *p.3, l.26* — "than the general 2-way" should be "than general 2-way".
- [REF1] *p.4, Definition 2.1.* — The notation using the underscore symbol $_$ is somehow unusual and should be explained.
- [REF1] *p.4, l.-1* — "family, that of regular languages, [13,14]" should be something like "family, namely the family of regular languages (see, for example, [13,14])".
- [REF1] *p.5, l.16* — "is said trivial" should be "is called trivial".
- [REF1] *p.5, l.27* — "Trivially, deterministic implies unambiguous." should be something like "Trivially, determinism implies unambiguity." or "Trivially, a deterministic finite automaton is unambiguous as well."
- [REF1] *p.5, l.33* — "The the controlled composition" should be "The controlled composition".
- [REF1] *p.6, l.9* — "nonempty" should be "non-empty".
- [REF1] *p.7, l.1* — "... no 1-way transducer accept it" should be something like "... no 1-way transducer can accept it"
- [REF1] *p.7, l.2* — "with A unambiguous" should be "with A being unambiguous"
- [REF1] *p.7, l.2-3* — "may accept it" should be "may accept UMULT"
- [REF1] *p.7, l.11* — " $(q_\ell, p + d)$ " probably should be " $(q_{\ell+1}, p + d)$ ".
- [REF1] *p.7, l.22* — "langage" should be "language".
- [REF1] *p.7, l.-7* — You should explain that $\phi(t)$ is a subset of $\phi_1(t)$. Is it clear that t belongs to δ_1 ?
- [REF1] *p.7, l.-5* — ... should be ...
- [REF1] *p.7, l.-4* — "occuring" should be "occurring".
- [REF1] *p.7, l.-2* — Remove the commas in " $r_0@, \dots, @r_{k-1}$ ". Replace ... by ...
- [REF1] *p.7, l.-1* — "non trivial" should be "non-trivial".

- [REF1] p. 7, l. -1 — Remove the comma in between "... run, which ...".
- [REF1] p. 8, l. 1 — "... state q' , in the last ..." Remove the comma.
- [REF1] p. 8, l. 2 — "... restless step i.e., ..." should be "... restless step, i.e., ..."
- [REF1] p. 8, l. 2 — "This require" should be "This requires".
- [REF1] p. 8, l. 7 — "There exists" should be "There exist".
- [REF1] p. 8, l. 10 — You should indicate in the union sign of the formula that q is accepting.
- [REF1] p. 8, l. 13 — Remove the line feed after "where".
- [REF1] p. 8, l. 18 — "one-wayness and sweepingness" sounds strange. Maybe "preserves the property of being one-way and sweeping" sounds better.
- [REF1] p. 9, l. 2 — "capture all the family" should be "capture the whole family".
- [REF1] p. 9, l. 10 — "Hadamard-product" should be "Hadamard product".
- [REF1] p. 9, l. 12 — "Hadamard-star" should be "Hadamard star".
- [REF1] p. 9, l. 18 — "However the H -star" should be "However, the H -star".
- [REF1] p. 9, l. -8 — " R_i s and S_i s" should be " R_i 's and S_i 's".
- [REF1] p. 10, l. -11 — "We claims:" should be "We claim:".
- [REF1] p. 10, l. -3 — You should add " (u) " after $S_{i_n}^*$.
- [REF1] p. 12, l. 9 — "Beware that ..." should be "Observe that ...".
- [REF1] p. 13, l. 4 — Explain "because \emptyset ."
- [REF1] p. 13, l. 9 — What do you mean by "computable rat-expression"? Is it effectively constructible?
- [REF1] p. 13, l. 15 — The notion of M' is quite uncommon. What about $M' = i - s \mid \dots \cup p + i - s \mid \dots$?
- [REF1] p. 13, l. 26 — "euclidian division" should be "Euclidean division".
- [REF1] p. 14, l. 20 — "... by proving intermediate ..." should be "... by proving an intermediate ..."
- [REF1] p. 15, l. 18 — "We pose ..." should be "We set ..."
- [REF1] p. 16, l. 3 — "greater common divisor" should be "greatest common divisor".
- [REF1] p. 16, l. 13 — "... since it is strongly ..." should be "... since this fact is strongly ..."
- [REF1] p. 16, l. 15 "**semi-linear forms**" — You never explain what semi-linear forms are.
- [REF1] p. 16, l. 16 — "developped" should be "developed".
- [REF1] p. 16, l. 17 — "amoung" should be "among".
- [REF1] p. 16, l. 18 — "... k -valued, sweeping." should be "... k -valued, or sweeping."
- [REF1] p. 16, l. 20 — "Here we show ..." should be "Here, we show ..."
- [REF1] p. 16, l. 25 — Remove the comma in "transducers, a production".
- [REF1] p. 17, l. -7 — "Occurences" should be "Occurrences".
- [REF1] p. 18, l. 8-9 — "... from previous study" should be "... from our previous study"
- [REF1] p. 18, l. -13 — What do you mean by $Z=R, S$? $Z \in R, S$?
- [REF1] p. 19, l. 6 — You should write $p(w)$ instead of p . Explain more detailed how Lemma 3.2. (which X ?) preserves the period (which?) and show that $p(w)$ is a period of $R(w)$.
- [REF1] p. 20, l. 18 — "... by sweeping transducer over ..." should be "... by sweeping transducers over ..."

- [REF1] p.20, l.-7 — *"is rational therefore Hadamard" should be "is rational, and therefore Hadamard,"*
- [REF1] p.20, l.-2 — *The hyphenation of "componentwise" is not correct.*
- [REF1] p.21, l.1 — *Remove the , in "transducer, even" and remove the hyphen in "unary-input".*
- [REF1] p.21, l.3 — *"sweeping transducer" should be "sweeping transducers"*
- [REF1] p.21, l.-13 — *You may refer to Lemma 2.7.*
- [REF1] p.21, l.-12 — *"that this is no loss" should be "that it is no loss"*
- [REF1] p.21, l.-9 — *You should again make clear what you mean by single-valuedness.*
- [REF1] p.22, l.4 — *Maybe b for border is a bad choice since b is also an input symbol. Another letter may be better suited.*
- [REF1] p.22, l.-13 — *"... that it is recognizable" What is recognizable?*
- [REF1] p.22, l.-10 — *Explain why the transducer "should also check that the input belongs to $D_{q,b,q'}$."*
- [REF1] p.22, l.-8 — *Give a precise definition of N .*
- [REF1] p.22, l.-7 — *Explain why it is easy to see that PREFIX and SUFFIX are functional. Owing to Def. 2.4 the output of a rational language is possible.*
- [REF1] p.23, l.15 — *"number of a or the number of b" should be "number of a's or the number of b's".*
- [REF1] p.23, l.15-16 — *"... is linearly bounded in N " Explain why this should be the case.*
- [REF1] p.23, l.-7 — *The hyphenation of "componentwise" is not correct.*
- [REF1] p.23, l.-3 — *You should define ID formally.*
- [REF1] p.24, l.3 — *"... is the hypothesis asking ..." Check the English of the sentence.*
-

4.2 Referee 2

[REF2] « Throughout the paper: The English and typesetting needs to be fixed. Several missing articles ("the"), missing final -s's, missing commas (before and after "i.e.", after "resp.", etc.), unnecessary hyphens, missing hyphens. Please have a native English speaker and perhaps a typesetter fix these mistakes. »

[REF2] p.3 l+6 — "sweeping transducers have ... than" -> "sweeping transducers are weaker than"

[REF2] p.4 l+9: — Concatenation is represented with a product-dot, when a few lines above it was represented without a product-dot.

[REF2] p.7 l+11: — $q_i - - > q_{i+1}$

[REF2] p.8 par-3: — bad line break

[REF2] Prop. 2.11, proof, l+1: — This sentence is a bit unclear. It took me time to understand what "the form given in the proposition" is (I thought it referred to $HAD(\Sigma * x\Delta^*)$).

[REF2] p.10, l-5: — In the expression at the start of the line, we need a pair of brackets around the subexpression which is being Kleene-starred (*).

[REF2] p.10 l-3: — In the expression at the end of the line, we need "(u)" next to each relation symbol.

[REF2] p.10 l-2: — Some more explanation will be helpful here. Just saying "by commutativity" is rather too dense.

[REF2] p.12 l+7: — converts -> corresponds

[REF2] p.13 l+4: — I believe the last copy of \emptyset should be $p\mathbb{N} \neq \emptyset$.

[REF2] Lemma 3.10, proof, l+1: — greater -> greatest

[REF2] Section 4, l+6: — developped -> developed

[REF2] Section 4, l+7: — amoung -> among

[REF2] Section 4.2, l+4: — identify Δ to -> identify Δ with

[REF2] p.20, par.+3: — Bad line break

[REF2] p.21 l+13: — output-unary -> input-unary

[REF2] Fig.5 caption: — else -> otherwise

[REF2] p.22 l-13: — recognizable means rational?

[REF2] p.22 l-3: — the notation $|v|_b$ is used without it having been introduced

[REF2] Sections 3.2 and 3.3: — Most of the results are much easier to prove if one thinks in terms of the unary DFAs that correspond to the sets involved. Perhaps a reference to them is appropriate.

[REF2] p.6: — To me, it sounds a bit strange to talk of a relation being "accepted" by a transducer. It seems more natural to talk of a relation being "generated".
