On relations accepted by two-way unary nondeterministic finite transducers

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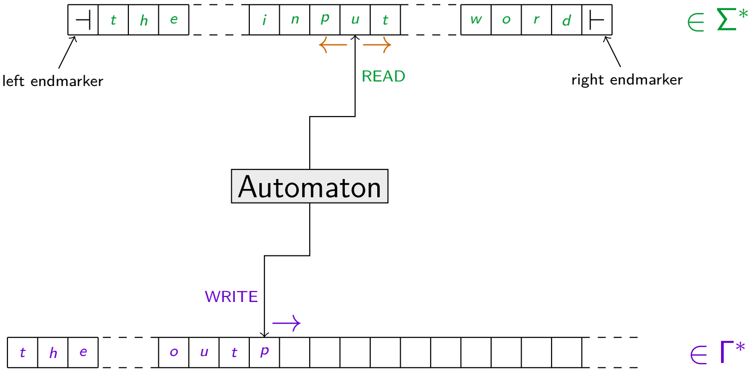
Two-way finite transducers



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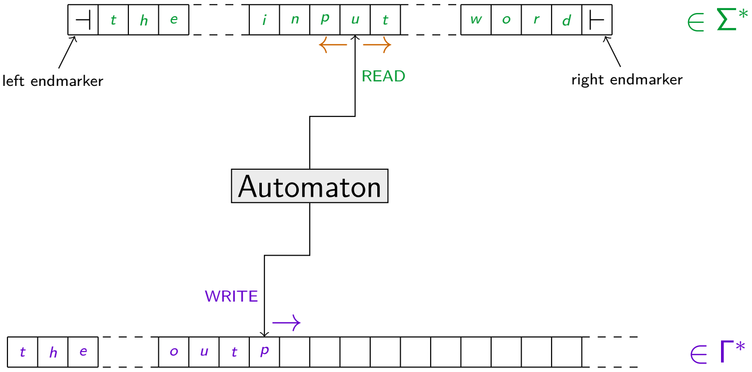
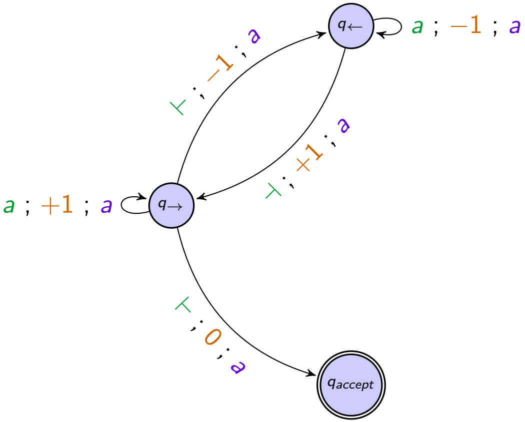


$$\delta \subset Q \times \Sigma \times \{-1, 0, +1\} \times Q \times \Gamma^*$$

Example

$$\Sigma = \Gamma = \{a\}$$



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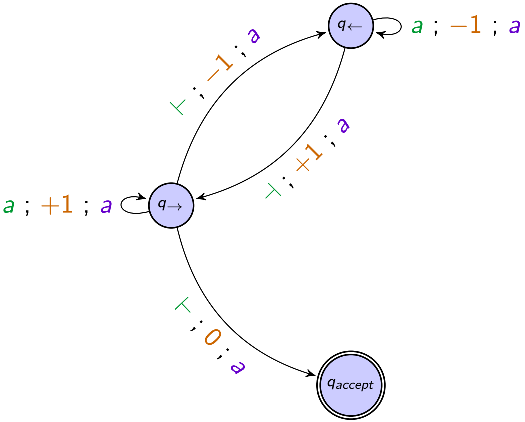

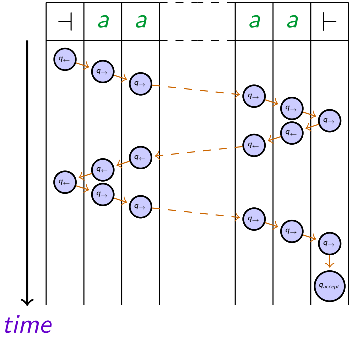

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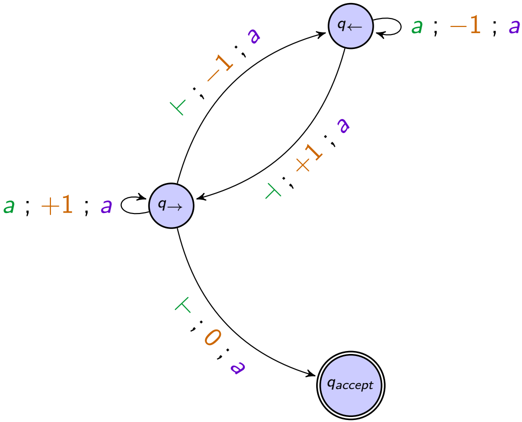

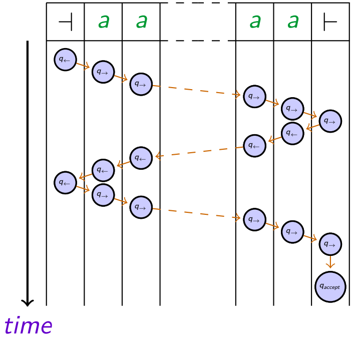

Non-rational accepted relation: $\mathcal{R} = \left\{ (a^n, a^{(2k+1)n}),\ n, k \in \mathbb{N} \right\}$.

Relations

Two-way transducers define binary relations (subsets of $\Sigma^* \times \Gamma^*$).

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Given such a relation $\mathcal{R}$, we represent it as a formal serie:

$$\tau = \sum_{w \in \Sigma^*} \alpha_w \cdot w \qquad \tau(w) = \alpha_w = \{v \in \Gamma^* \mid (w, v) \in \mathcal{R}\}$$

Rational series

Rational series of $\mathbb{K}\langle\langle M \rangle\rangle$:

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- closed under sum,

Sum

$$(\sigma + \tau)(w) = \sigma(w) + \tau(w)$$

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- closed under sum,
- Cauchy product

Cauchy Product

$$(\sigma \times \tau)(w) = \sum_{w = w_1 \cdot w_2} \sigma(w_1) \cdot \tau(w_2)$$

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- contains polynomial,
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- Cauchy product
- and Kleene star.

Kleene Star

$$(\sigma^*)(w) = \sum_{w = w_1 \cdot w_2 \cdots w_r} \sigma(w_1)\sigma(w_2)\ldots\sigma(w_r)$$

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Theorem

One-way transducers accept exactly $RAT(\Gamma^*)\langle\langle \Sigma^* \rangle\rangle$.

Known results

Theorem (Engelfriet, Hoogeboom, 2001)

- ***deterministic case***: *two-way transducers accept exactly the class of MSO-definable functions.*

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$$\boxed{\mathcal{T} = \{(w, w \cdot w) \mid w \in \Sigma^*\}}$$

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- ***deterministic case***: *two-way transducers accept exactly the class of MSO-definable functions.*
- ***nondeterministic case***: *the class of MSO-definable transductions and the class of relations accepted by two-way transducers are incomparable.*

Known results

Theorem (Filiot, Gauwin, Reynier, Servais, 2013)

It is ***decidable*** *whether some relation accepted by two-way transducer is accepted by some one-way transducer.*

$\rightarrow$ *construction of equivalent one-way transducer, whenever one exists.*

Unary case - our result

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Theorem

$\tau : \Sigma^{\mathbb{N}} \to 2^{\Gamma^{\mathbb{N}}}$ is accepted by a two-way transducer
if and only if
there exists finitely many rational series α_i and β_i such that

$$\forall n \quad \tau(a^n) \;=\; \bigcup_i \;\; (\alpha_i(a^n) \;\cdot\; \beta_i(a^n)^*)$$

Analogy with Probabilistic Automata

Theorem (Anselmo,Bertoni,1994)

Acceptation probability of two-way finite automata is of the form:

$$\tau(w) = \alpha(w) \times \frac{1}{\beta(w)}$$

where α and β are rational series of $\mathbb{Q}\langle\langle\Sigma^*\rangle\rangle$.

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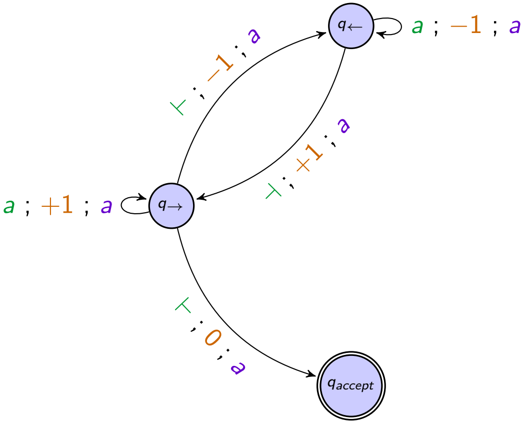

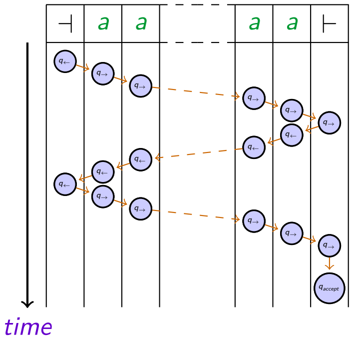

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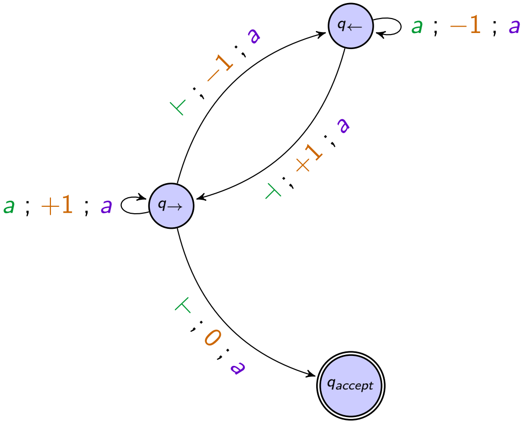

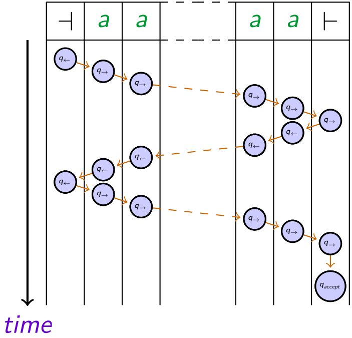

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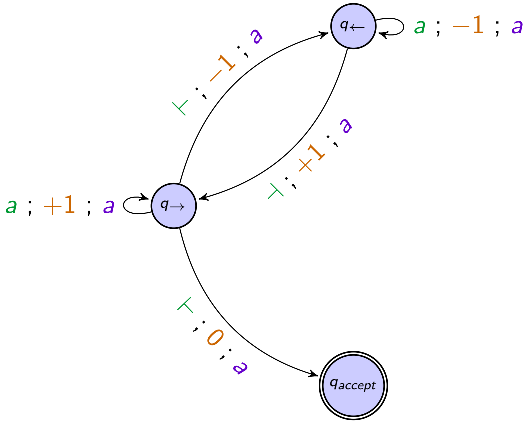

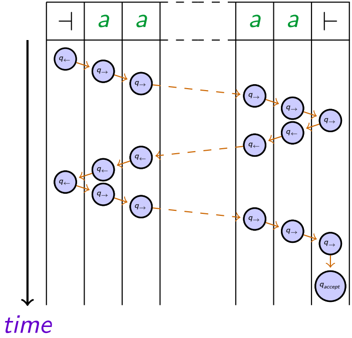

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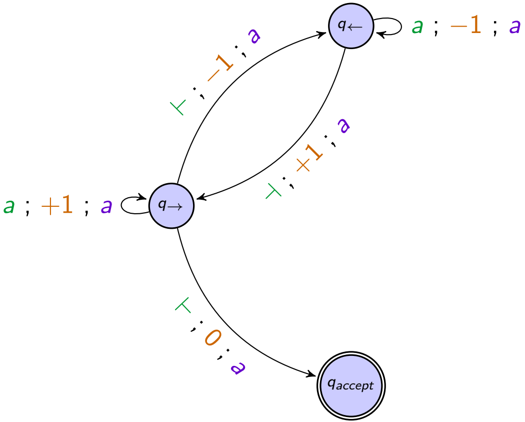

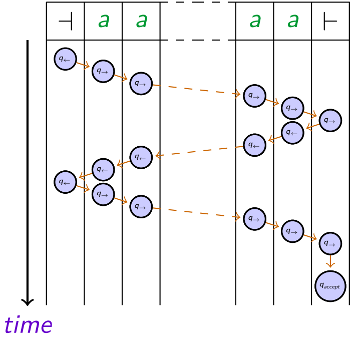

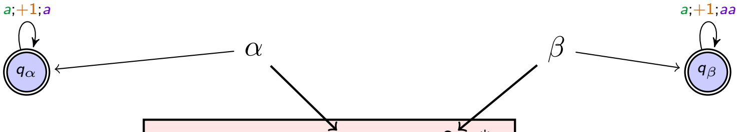

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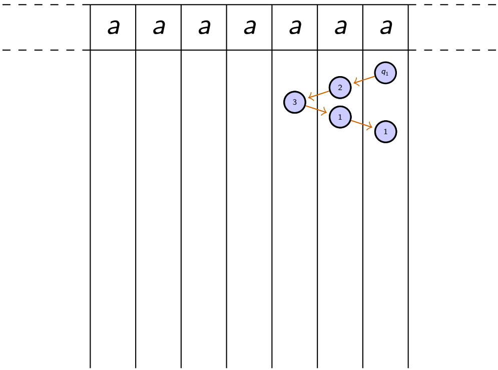

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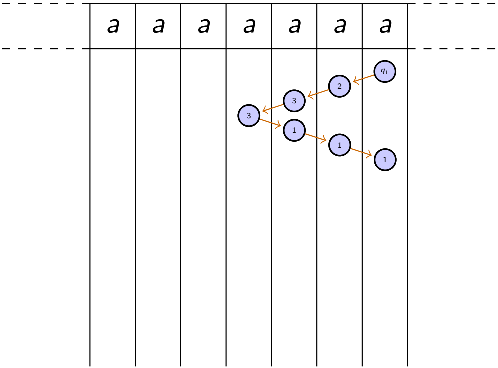

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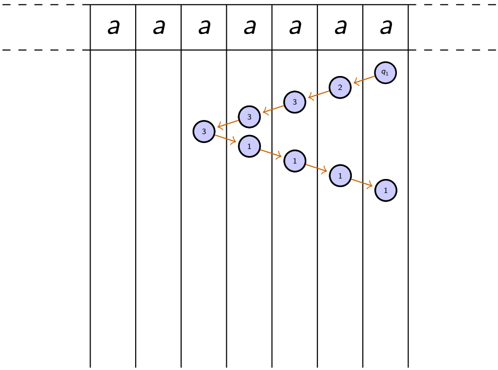

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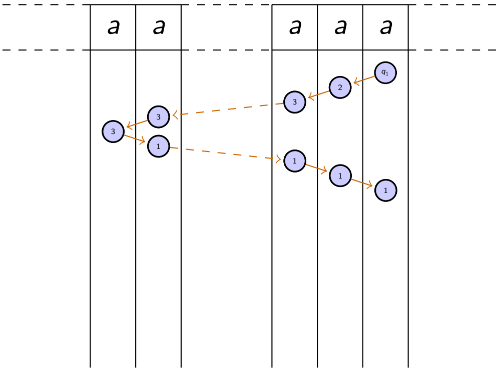

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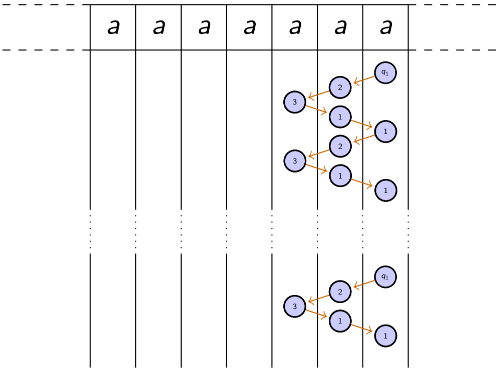

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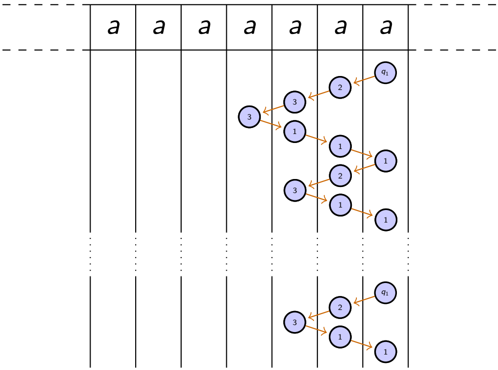

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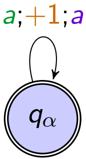

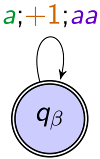

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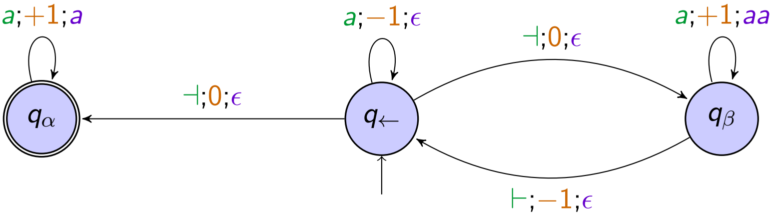

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Do you have any questions?